Nutational Stability of an Asymmetric Dual-Spin Spacecraft

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This paper employs an energy-sink approach to establish the closed form nutational stability criterion and the time constant of an asymmetric dual-spin spacecraft. The system under consideration consists of an asymmetric platform and a symmetric rotor. Each of these two components contains a damper. In such a system, the rotor damper is driven by harmonic forcing functions of two different frequencies. Recognizing this phenomenon, it is demonstrated through analysis and several specific numerical examples that the energy-sink predictions correlate extremely well with the system time simulation results.

Nomenclature

$\boldsymbol{B}$	$= 3 \times 3$ transformation matrix, Eq. (2)
$\boldsymbol{c}^{\beta}$	$= 3 \times 3$ diagonal damping matrix of D_R, Eq. (9)
c^{γ}	= damping constant of D_P, Eq. (10)
D_P	= platform damper, Fig. 1
D_R	= rotor damper, Fig. 1
$[\boldsymbol{d}]^T = (\boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{d}_3)$	= unit vector array fixed in D_R, Eq. (3)
$\boldsymbol{E}$	$= 3 \times 3$ identity matrix
$\boldsymbol{E}^i$	$= i^{\text{th}}$ column of $\boldsymbol{E}$
$\dot{E}_P, \dot{E}_R$	= energy dissipation rate in P and R, respectively Eqs. (36) and (38)
$\boldsymbol{E}^{12}$	= a 3×2 matrix, Eq. (9)
F^{γ}	= torque on D_P, Eq. (10)
$\boldsymbol{H}^C$	$= 3 \times 1$ spacecraft angular momentum matrix about its mass center in $[\boldsymbol{p}]$, Eq. (8)
h_0	= nominal angular momentum, Eq. (15)
I_1, I_2, I_3	= principal moments of inertia about $\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3$ respectively of the spacecraft with respect to its mass center when $\beta = 0$, Eq. (15)
$\boldsymbol{I}^d$	$= 3 \times 3$ diagonal inertia matrix of D_R with respect to its mass center in $[\boldsymbol{d}]$, Eq. (8)
$\boldsymbol{I}^r$	$= 3 \times 3$ diagonal inertia matrix of R with respect to its mass center in $[\boldsymbol{r}]$, Eq. (8)
$\boldsymbol{I}^*$	$= 3 \times 3$ inertia matrix of the system for its mass center excluding $\boldsymbol{I}^d$ and $\boldsymbol{I}^r$, Eq. (8)
J	= spin inertia of D_P about $\boldsymbol{p}_2$ axis, Eq. (7c)
$\boldsymbol{k}^{\beta}$	$= 3 \times 3$ diagonal stiffness matrix of D_R, Eq. (9)
k^{γ}	= spring constant of D_P, Eq. (10)
$\boldsymbol{P}$	= platform, Fig. 1
$[\boldsymbol{p}]^T = (\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3)$	= unit vector array fixed in P, Fig. 1
p, p'	= frequency of D_P and D_R, respectively, Eq. (23)
p_*	= stiffened rotor damper frequency, Eq. (30)
R	= rotor, Fig. 1
$[\boldsymbol{r}]^T (\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)$	= unit vector array fixed in R, Fig. 1
$\boldsymbol{T}^P$	$= 3 \times 1$ external torque matrix, Eq. (7a)
$\boldsymbol{T}^d$	$= 3 \times 1$ torque matrix of D_R, Eq. (7b)
T_C	= time constant, Eq. (45)
β_1, β_2	= tilting angle of D_R about $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ respectively, Eq. (4), Fig. 1
γ	= twisting angle of D_P about $\boldsymbol{p}_2$, Eq. (7c), Fig. 1
ζ, ζ'	= damping ratio of D_P and D_R respectively, Eq. (23)
λ	= platform driving frequency, Eq. (19)
λ_P, λ_R	= normalized angular rates, Eq. (44)
σ	= rotor speed relative to P, a constant, Eq. (2)
σ_1, σ_2	= rotor driving frequencies, Eq. (30)
τ, τ'	= interval for averaging platform and rotor energy dissipation rate, respectively, Eqs. (37) and (39)
ψ	= angle between $\boldsymbol{r}_1$ and $\boldsymbol{p}_1$, Fig. 1
Ω	= nominal platform rate about $\boldsymbol{p}_3$, Eq. (11)
$\boldsymbol{\omega}^P$	$= 3 \times 1$ inertial platform rate in $[\boldsymbol{p}]$, Eq. (6)
ω_0	= initial value of ω_2, Eq. (18)

Special Symbols

$$\tilde{v} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \quad \text{with } v = \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix}$$

$$\min(a, b) = \begin{cases} a & \text{if } a < b \\ a \text{ or } b & \text{if } a = b \\ b & \text{if } a > b \end{cases}$$

$(\ )^T, [\ ]^T =$ Transpose of $(\ ), [\ ]$

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Introduction

THE so-called "dual-spin" stabilization concept was first announced in the open literature by Landon and Stewart[1] in 1964. It was later expanded by Iorillo,[2] and formalized by Likins[3] and Mingori[4] through Routh and Floquet analyses for some specific types of energy dissipation mechanisms. Among the results presented in Ref. 3, there is also a general nutational stability criterion for dual-spin spacecraft. This criterion was obtained through an approximate energy-sink analysis and has a compact and simple form. For the convenience of the subsequent discussions throughout this paper, it will be referred to as the "familiar stability condition."

This condition has been applied in the past to many symmetric dual-spin spacecraft. For an asymmetric dual-spin system with a despun platform, one extension of the familiar stability condition is that rotor damping is stabilizing if the rotor spin inertia I_s is greater than the arithmetic mean of the vehicle transverse inertias, i.e., $\frac{1}{2}(I_1+I_2)$. Cherchas and Hughes[5] showed, through an eigenvalue analysis for a particular spacecraft configuration, that the corresponding condition is $I_s > (I_1 I_2)^{1/2}$, and suggested that the discrepancy might be due to the inherently approximate nature of the energy sink approach employed in Ref. 3. Using a modified energy-sink procedure, Spencer[6] obtained results which support the importance of the geometric mean, i.e., $(I_1 I_2)^{1/2}$. The crucial modification proposed by Spencer is to include the time-varying "generalized nutation rates" in the averaging process of the platform and rotor energy dissipation rates. Reference 3 normalizes the energy dissipation rates with respect to two parameters which are constants by definition and happen to be the nutation rates for a symmetric dual-spin system. This approach, adopted in the present paper, considerably simplifies the analytical effort in deriving closed form stabiliby criteria for a class of asymmetric dual-spin systems whose geometric mean of the transverse inertias does not differ appreciably from the arithmetic mean. The difference between the two transverse inertias, however, can be significant. It is worth noting that most of the real, modern dual-spin spacecraft belong to this class.

This paper treats the system with the rotor damper being driven by the body motion which can be identified as harmonic forcing functions of two different frequencies in computing the energy dissipation rates. Substitution of these energy dissipation rates into the familiar stability condition yields closed-form representations of the nutational stability criterion and the time constant. The predictions based on these results correlate extremely well with the system time simulations of several numerical examples of practical interest. The spacecraft system under consideration is described next.

Spacecraft System Description

An idealized dual-spin spacecraft is shown in Fig. 1. $\boldsymbol{P}$ denotes the rigid asymmetric platform; $\boldsymbol{R}$ represents the rigid symmetric rotor. The energy-loss mechanism on the platform is idealized by a wheel damper, $\boldsymbol{D_R}$, which can be used to simulate the energy loss effect in a fluid damper. Its alignment with p_2 axis is arbitrarily chosen. The energy-loss mechanism on the rotor is idealized by a two-degree-of-freedom, tilting ring damper, $\boldsymbol{D_R}$. The torsional motion of the platform damper and the tilting motion of the rotor damper are resisted by linear spring and viscous damping torques. Nonlinear damper characteristics can result in very interesting and complex phenomena[6-8] where certain nonlinearities were assumed. Establishing and modeling the exact physical nature of real nonlinear damping mechanisms is an area worth further study.

Let $[p]^T = (p_1, p_2, p_3)$ be a unit vector array fixed in the platform; $[r]$ a unit array fixed in the rotor with $r_3 = p_3$. The angle between r_1 and p_1 is called ψ. The rotor speed about r_3 or p_3 relative to the platform is maintained at a constant σ. The transformation from $[p]$ to $[r]$ is defined by the following relationship

$$[r] = B[p] \tag{1}$$

where

$$B = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

and $\psi = \sigma t$.

In what follows, the tilting motion of the ring damper (D_R) relative to the rotor (R) is assumed infinitesimal as is the case in many elastic systems. Terms involving the tilting coordinates and their time derivatives to the second or higher orders will be ignored in the development of the coordinate transformation matrices and the system dynamical equations. This simplification facilitates the subsequent analytical work without sacrificing the salient dynamic features of the spacecraft under consideration, and will lead to a correct set of fully linearized equations for the stability analysis. Let $[d]$ be a unit vector array fixed in D_R, and β_1 and β_2 be the infinitesimal tilting angles of the D_R relative to R. Assuming that initially $[d]$ coincides with $[r]$, a rotation of β_1 followed by a rotation of β_2 about the new d_2 (Fig. 1) leads to the following relationship

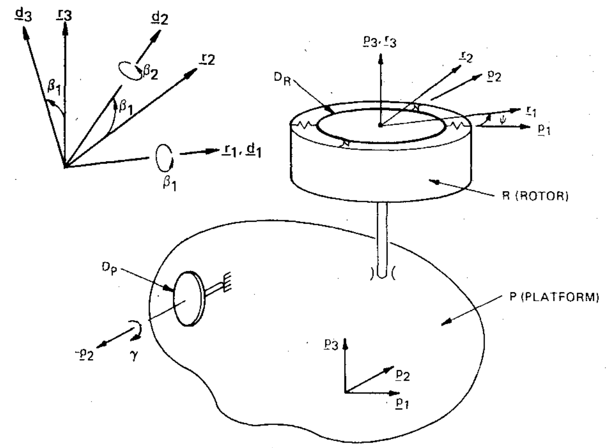


Fig. 1 Idealized dual-spin spacecraft model.

$$[d] = (E - \tilde{\beta})[r] \tag{3}$$

where E is the identity matrix, and

$$\beta^T = (\beta_1, \beta_2, 0) \tag{4}$$

and $\tilde{\beta}$ is defined in the same manner as $\tilde{v}$, in the Nomenclature; i.e.

$$\tilde{\beta} = \begin{bmatrix} 0 & 0 & \beta_2 \\ 0 & 0 & -\beta_1 \\ -\beta_2 & \beta_1 & 0 \end{bmatrix} \tag{5}$$

Furthermore, the twisting angle between D_P and P about p_2 is called γ. The inertial angular velocity of P is defined as

$$\omega^P = [p]^T \omega^P \tag{6}$$

Using the above coordinates and the transformation matrices, one can now proceed to derive the equations of motion of the system.

Equations of Motion

The spacecraft attitude dynamical equations in the matrix form obtained through the Newton-Euler formulation are given below without proof

$$a^\omega \dot{\omega}^P + a^\beta \ddot{\beta} + a^\gamma \ddot{\gamma} = T^P - a^N \tag{7a}$$

$$[E^{12}]^T (b^\omega \dot{\omega}^P + b^\beta \ddot{\beta}) = [E^{12}]^T (T^d - b^N) \tag{7b}$$

$$J([E^2]^T \dot{\omega}^P + \ddot{\gamma}) = F^\gamma \tag{7c}$$

where

$$a^\omega = I^* + B^T (I^r + J^r) B \tag{8a}$$

$$a^\beta = B^T J^r \tag{8b}$$

$$a^\gamma = J E^2 \tag{8c}$$

$$a^{\sigma}=B^T(I^r+J^r)BE^3 \tag{8d}$$

$$J^r=I^d+\tilde{\beta}I^d-I^d\tilde{\beta} \tag{8e}$$

$$a^N=\dot{a}^{\omega}\omega^P+\dot{a}^{\beta}\dot{\beta}+\dot{a}^{\gamma}\dot{\gamma}+\dot{a}^{\sigma}\sigma+\tilde{\omega}^P H^C \tag{8f}$$

$$H^C=a^{\omega}\omega^P+a^{\beta}\dot{\beta}+a^{\gamma}\dot{\gamma}+a^{\sigma}\sigma \tag{8g}$$

and

$$b^{\omega}=J^rB \tag{9a}$$

$$b^{\beta}=J^r \tag{9b}$$

$$T^d=-(c^{\beta}\dot{\beta}+k^{\beta}\beta) \tag{9c}$$

$$b^N=J^r\dot{B}(\omega^P+\sigma E^3)+\tilde{\omega}^r J^r\omega^d+\dot{J}^r\omega^d \tag{9d}$$

$$\omega^d=\dot{\beta}+B(\omega^P+\sigma E^3) \tag{9e}$$

$$\omega^r=B(\omega^P+\sigma E^3) \tag{9f}$$

$$E^{12^T}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix} \tag{9g}$$

$$F^{\gamma}=-(c^{\gamma}\dot{\gamma}+k^{\gamma}\gamma) \tag{10}$$

Physically significant parameters not defined in the preceding section are described under "Nomenclature," The matrix $[E^{12}]^T$ is introduced to drop the third equation in Eq. (7b), which is not of interest here.

Energy Sink Analysis

Equation (7) contains six differential equations. These equations can be solved only through numerical intergrations on a computer. The stability of the motion under question is in the neighborhood of a nominal spin† of the spacecraft. Linearization of these differential equations with respect to small variation coordinates about the nominal spin will lead to a set of linear differential equations with periodic coefficients.

The stability question of a set of linear differential equations with periodic coefficients can be answered using a Floquet analysis similar to that presented in Refs. 4 and 9, or the formal averaging method which Vigneron[10] applied to a symmetric dual-spin system. The Floquet analysis, however, is a numerical process which does not provide the closed-form stability criterion. Furthermore, it does not yield information concerning the time constant of the spacecraft nutational decay or growth. It is possible, though, to remove the periodic coefficients through a coordinate transformation of the β_i's[5,11] or directly defining the β_i's in the platform fixed reference frame.[12] The resulting linear differential equations with constant coefficient can then be analyzed through Routh array or the theorems advanced by Mingori[12,13] to establish the stability criteria for the system. The question concerning the nutational time constant has to be determined by the eigenvalues of the system. Because of the high dimension of the problem usually encountered in practice, closed-form solutions of the eigenvalues are not possible, and a numerical procedure utilizing a computer must be employed.

The energy-sink approach, on the other hand, can provide closed-form representation of the stability criterion, as well as the nutational time constant. The approach, although approximate in nature, has been proven to be an invaluable design tool in the past as was applied to many symmetrical dual-spin spacecraft.[7,8] Its application to asymmetric dual-spin spacecraft is more involved, because the motion of such a system is much more complex. This paper treats the system with the rotor damper being driven by the body motion which can be identified as harmonic forcing functions of two different frequencies in computing the energy dissipation rates. The platform damper is still driven by a harmonic forcing function of a single frequency, as is the case in a symmetric system. The analytical procedure employed in the present paper is briefly described next.

†For the nominal spin, the spacecraft spin axis (p_3 or r_3) is aligned with the angular momentum vector.

Following the general procedures of the energy-sink analysis, the three scalar equations describing the spacecraft attitude motion were solved by initially ignoring D_P and D_R. These solutions were, in turn, considered as the forcing functions to drive D_P and D_R to dissipate energy. The energy dissipation rates were then determined and substituted into the familiar stability condition[3] to yield closed-form representations of the stability criterion, as well as the time constant. The energy-sink predictions were finally checked against the partially linearized system simulation results for two specific spacecraft systems with transverse inertia ratio (I_1/I_2) at 1.5 and 1.7.

Assuming a torque-free environment, linearization of the differential equations in Eq. (7a) with respect to small variational coordinates about the nominal state, i.e.,

$$\omega_3^P=\Omega \text{ (constant)}, \tag{11a}$$

$$\omega_1^P=\omega_2^P=\beta_1=\beta_2=\gamma=0 \tag{11b}$$

leads to the following scalar equations, upon suppressing the subsystem coordinates γ and β_i's and their time derivatives

$$\dot{\omega}_1+\lambda_1\omega_2=0 \tag{12}$$

$$\dot{\omega}_2-\lambda_2\omega_1=0 \tag{13}$$

$$\dot{\omega}_3=0 \tag{14}$$

where

$$\omega=\omega^P-\Omega E^3 \tag{15a}$$

$$\lambda_1=\frac{h_0-I_2\Omega}{I_1};\quad \lambda_2=\frac{h_0-I_1\Omega}{I_2} \tag{15b}$$

$$h_0=I_3\Omega+(I_{33}^r+I_{33}^d)\sigma \tag{15c}$$

In Eq. (15), h_0 is the nominal spacecraft angular momentum, and I_1, I_2, I_3 are the principal moments of inertia of the rigid dual-spin spacecraft system (i.e., $P+D_P+R+D_R$) for its mass center. The solutions for Eqs. (12) and (13) are given in Ref. 3 and repeated below

$$\omega_1=-\omega_0(\lambda_1/\lambda_2)^{1/2}\sin\lambda t \tag{16}$$

$$\omega_2=\omega_0\cos\lambda t \tag{17}$$

for the initial conditions

$$\omega_1(0)=0;\omega_2(0)=\omega_0 \tag{18}$$

and with

$$\lambda=(\lambda_1\lambda_2)^{1/2} \tag{19}$$

Using the assumption that $I_{11}^d=I_{22}^d=\frac{1}{2}I_{33}^d$, linearization of Eqs. (7b) and (7c) with respect to ω_1, ω_2, ω_3, β_1, β_2, and γ yields the following

$$\ddot{\beta}_1+2p'\zeta'\dot{\beta}_1+[p'^2+(\Omega+\sigma)^2]\beta_1$$
$$=-\cos\psi[\dot{\omega}_1+(\Omega+2\sigma)\omega_2]-\sin\psi[\dot{\omega}_2-(\Omega+2\sigma)\omega_1] \tag{20}$$

$$\ddot{\beta}_2+2p'\zeta'\dot{\beta}_2+[p'^2+(\Omega+\sigma)^2]\beta_2$$
$$=\sin\psi[\dot{\omega}_1+(\Omega+2\sigma)\omega_2]-\cos\psi[\dot{\omega}_2-(\Omega+2\sigma)\omega_1] \tag{21}$$

$$\ddot{\gamma}+2p\zeta\dot{\gamma}+p^2\gamma=-\dot{\omega}_2 \tag{22}$$

where p', ζ' (or p, ζ) are the frequency and damping ratio, respectively, for D_R (or D_P); i.e.

$$p'^2=k^\beta{}_{11}/I^d{}_{11}; \qquad 2p'\zeta'=c^\beta{}_{11}/I^d{}_{11}$$
$$p^2=k^\gamma/J; \qquad 2p\zeta=c^\gamma/J \tag{23}$$

It should be noted that dampers of a more general nature than the ones under consideration in the present paper can lead to equations more complex than Eqs. (20) through (22).

Substitution of Eqs. (16) and (17) into the above equations leads to the following equations

$$\ddot{\beta}_1+2p'\zeta'\dot{\beta}_1+[p'^2+(\Omega+\sigma)^2]\beta_1$$
$$=\omega_0[a\cos\sigma t\cos\lambda t+b\sin\sigma t\sin\lambda t] \tag{24}$$

$$\ddot{\beta}_2+2p'\zeta'\dot{\beta}_2+[p'^2+(\Omega+\sigma)^2]\beta_2$$
$$=\omega_0[-a\sin\sigma t\cos\lambda t+b\cos\sigma t\sin\lambda t] \tag{25}$$

$$\ddot{\gamma}+2p\zeta\dot{\gamma}+p^2\gamma=\omega_0\lambda\sin\lambda t \tag{26}$$

where

$$a=-(\Omega+2\sigma-\lambda_1); \qquad b=-[(\Omega+2\sigma)(\lambda_1/\lambda_2)^{1/2}-\lambda] \tag{27}$$

One can rewrite Eqs. (24) and (25) as

$$\ddot{\beta}_1+2p'\zeta'\dot{\beta}_1+p_*^2\beta_1=\omega_0(a_1\cos\sigma_1 t+a_2\cos\sigma_2 t) \tag{28}$$

$$\ddot{\beta}_2+2p'\zeta'\dot{\beta}_2+p_*^2\beta_2=\omega_0(-a_1\sin\sigma_1 t-a_2\sin\sigma_2 t) \tag{29}$$

where

$$p_*^2=p'^2+(\Omega+\sigma)^2$$
$$a_1=\tfrac{1}{2}(a-b); \qquad a_2=\tfrac{1}{2}(a+b) \tag{30}$$
$$\sigma_1=\sigma+\lambda; \qquad \sigma_2=\sigma-\lambda$$

It becomes apparent from Eq. (26) that the platform damper, D_P, is driven by the frequency λ, and, from Eqs. (28) and (29), that the rotor damper, D_R, is driven by harmonic functions of two different frequencies σ_1 and σ_2.

The steady-state solutions of Eqs. (26), (28), and (29) have the following forms

$$\gamma=\omega_0\gamma_0\sin(\lambda t+\phi_\gamma) \tag{31}$$

$$\beta_1=\omega_0[\beta_{11}\sin(\sigma_1 t+\phi_{11})+\beta_{12}\sin(\sigma_2 t+\phi_{12})] \tag{32}$$

$$\beta_2=\omega_0[\beta_{21}\sin(\sigma_1 t+\phi_{21})+\beta_{22}\sin(\sigma_2 t+\phi_{22})] \tag{33}$$

The amplitudes and phase angles of these solutions are given below

$$\gamma_0=\frac{\lambda}{[(2p\zeta\lambda)^2+(p^2-\lambda^2)^2]^{1/2}} \tag{34}$$

$$\phi_\gamma=\tan^{-1}\frac{2p\zeta\lambda}{(\lambda^2-p^2)}$$

$$\beta_{11}=\beta_{21}=\frac{-a_1}{[(2p'\zeta'\sigma_1)^2+(p_*^2-\sigma_1^2)^2]^{1/2}}$$

$$\beta_{12}=\beta_{22}=\frac{-a_2}{[(2p'\zeta'\sigma_2)^2+(p_*^2-\sigma_2^2)^2]^{1/2}} \tag{35}$$

$$\phi_{21}=\tan^{-1}\frac{2p'\zeta'\sigma_1}{(\sigma_1^2-p_*^2)}; \quad \phi_{22}=\tan^{-1}\frac{2p'\zeta'\sigma_2}{(\sigma_2^2-p_*^2)}$$

$$\phi_{11}=\phi_{21}-\frac{\pi}{2}; \quad \phi_{12}=\phi_{22}-\frac{\pi}{2}$$

The energy dissipation rate in the platform is:

$$\dot{E}_P=\frac{1}{\tau}\int_0^\tau(-c^\gamma\dot{\gamma}^2)\,dt \tag{36}$$

It is convenient to choose

$$\tau=2\pi/|\lambda| \tag{37}$$

Similarly, the energy dissipation rate in the rotor is

$$\dot{E}_R=\frac{1}{\tau'}\int_0^{\tau'}[-c^\beta_{11}(\dot{\beta}_1^2+\dot{\beta}_2^2)]\,dt \tag{38}$$

Unlike the platform damper which is driven by one single frequency, λ, the rotor damper follows two different frequencies: σ_1 and σ_2. The inteval for integration τ' can be selected as

$$\tau'=N\frac{2\pi}{\min(|\sigma_1|,|\sigma_2|)} \tag{39}$$

where N is an arbitrary integer, as will be demonstrated in the numerical examples. $\min(|\sigma_1|,|\sigma_2|)$ is defined in the Nomenclature.

Substitution of Eqs. (31), (32), and (33) into Eqs. (36) and (38) leads to the following expressions

$$\dot{E}_P=-\tfrac{1}{2}\omega_0^2c^\gamma\lambda^2\gamma_0^2 \tag{40}$$

$$\dot{E}_R=-\omega_0^2\frac{c^\beta_{11}}{\tau'}\{[\beta_{11}^2\sigma_1^2+\beta_{12}{}^2\sigma_2^2]\tau'$$
$$+\frac{2\beta_{11}\beta_{12}\sigma_1\sigma_2}{\sigma_3}[\sin(\sigma_3\tau'+\phi_{13})-\sin\phi_{13}]\} \tag{41}$$

where

$$\sigma_3=\sigma_1-\sigma_2,\text{ and }\phi_{13}=\phi_{11}-\phi_{12} \tag{42}$$

These rates can now be used to establish the nutational stability criterion. Before we do this, it is important to review the following result, obtained in Ref. 3

$$\omega_0\dot{\omega}_0=\left(\frac{\dot{E}_P}{\lambda_P}+\frac{\dot{E}_R}{\lambda_R}\right)\frac{2h_0\lambda_2}{I_1^2\lambda_1+I_2^2\lambda_2} \tag{43}$$

The two new symbols, λ_P and λ_R, in the above equation are defined in the reference. They are repeated below

$$\lambda_P=h_0\frac{I_1\lambda_1+I_2\lambda_2}{I_1^2\lambda_1+I_2^2\lambda_2}-\Omega \tag{44a}$$

$$\lambda_R=h_0\frac{I_1\lambda_1+I_2\lambda_2}{I_1^2\lambda_1+I_2^2\lambda_2}-(\Omega+\sigma) \tag{44b}$$

It is worth noting that λ_P and λ_R are both constants by definition and happen to be the nutation rates in P and R in a symmetric system. The nutation rates in P and R of an asymmetric dual-spin spacecraft, as pointed out in Spencer's paper,[6] are time-varying quantities.

Substitution of Eqs. (40) and (41) into Eq. (43) provides the following expression for the nutational time constant, T_C

$$T_C^{-1} = \frac{2h_0\lambda_2}{I_1^2\lambda_1 + I_2^2\lambda_2}\left\{-\frac{c^\gamma\lambda^2\gamma_0^2}{2\lambda_P} - \frac{c^\beta_{11}}{\tau'\lambda_R}\left[(\beta_{11}^2\sigma_1^2+\beta_{12}^2\sigma_2^2)\tau' + \frac{2\beta_{11}\beta_{12}\sigma_1\sigma_2}{\sigma_3}(\sin[\sigma_3\tau'+\phi_{13}]-\sin\phi_{13})\right]\right\} \qquad (45)$$

The symbols in the above equation have all been previously defined. The familiar stability condition, i.e.

$$(\dot{E}_P/\lambda_P) + (\dot{E}_R/\lambda_R) < 0 \qquad (46)$$

has the following interpretation

$$-\frac{c^\gamma\lambda^2\gamma_0^2}{2\lambda_P} - \frac{c^\beta_{11}}{\tau'\lambda_R}\left\{(\beta_{11}^2\sigma_1^2+\beta_{12}^2\sigma_2^2)\tau' + \frac{2\beta_{11}\beta_{12}\sigma_1\sigma_2}{\sigma_3}[\sin(\sigma_3\tau'+\phi_{13}) \qquad (47)$$

$$-\sin\phi_{13}]\right\} \lesseqgtr 0 \quad \Longrightarrow \quad \begin{matrix}\text{Stability}\\ \text{Equilibrium}\\ \text{Instability}\end{matrix}$$

The application of these results will now be demonstrated in numerical examples.

Numerical Results

Case I

In this example, the basic RCA Satcom spacecraft parameters during transfer orbit were used; i.e., $I_1 = 2242.2$ lb-in.-sec^2, $I_2 = 1473.5$ lb-in.-sec^2, $I_3 = 2031.6$ lb-in.-sec^2, $h_0 = 1174$ lb-in.-sec^2, and $\Omega = 0.45029$ rad/sec. The rotor and platform damper parameters are selected as follows: $I^r_{11} = I^r_{22} = \frac{1}{2}I^r_{33} = 8.08249$ lb-in.-sec^2, $I^d_{11} = I^d_{22} = \frac{1}{2}I^d_{33} = 0.01723$ lb-in.-sec^2, (so that $\sigma = 16$ rad/sec and $\lambda = 0.15931$ rad/sec), $p' = 4$ rad/sec, $\zeta' = 0.05$, $J = 5.84$ lb-in.-sec^2, $c^\gamma = 0.436$ lb-in.-sec, and $k^\gamma = 0$. Note that, from Eq. (23), $p = 0$ because $k^\gamma = 0$, but $2p\zeta = c^\gamma/J \neq 0$.

In view of the fact that λ is considerably smaller than σ, τ' can then be chosen as $2\pi/\sigma$. Substitution of the above parameter values into Eqs. (45) and (47) indicates that the system is asymptotically stable and its decay time constant is 64.9 min. The nonlinear system described by Eq. (7) was programmed on an IBM 370 computer for time simulations. The time simulation result of the nutation angle, initially at 0.1 deg, is shown in Fig. 2. The decay time constant derived from the time simulation is around 65.9 min. This correlates remarkably well with the energy-sink prediction.

Case II

$I^r_{11} = I^r_{22} = \frac{1}{2}I^r_{33} = 7.92742$ lb-in.-sec^2, $I^d_{11} = I^d_{22} = \frac{1}{2}I^d_{33} = 0.1723$ lb-in.-sec^2, $\zeta' = 0.2$. All other parameters are the same as those defined in Case I. The energy sink analysis, Eq. (45), predicts for this case a growth time constant (so that the system is unstable) of 62.2 min. The time simulation result shown in Fig. 3, indicates a growth time constant of 61.7 min.

Case III

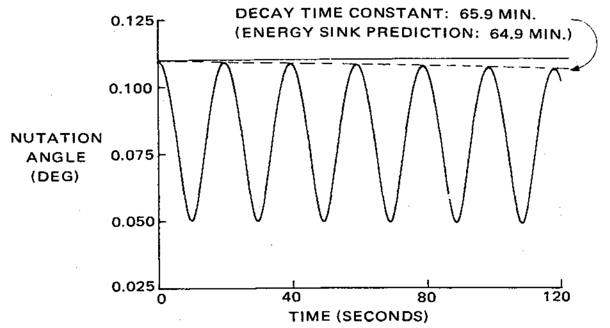
This case is identical to Case I, except that the initial nutation angle is 20 deg. The simulation result is shown in Fig. 4. The time constant derived from the time simulation is about 64.1 min. This is again very close to the predicted value of 64.9 min.

Case IV

In the above three cases, the value of λ is considerably smaller than that of σ. Here, we will study a case in which λ and σ are comparable in magnitude. The following parameters with units identical to those defined in Case I were chosen: $I_1 = 5000$, $I_2 = 3000$, $I_3 = 4000$, $h_0 = 1200$, $\Omega = 0$, $I^r_{11} = I^r_{22} = \frac{1}{2}I^r_{33} = 1399$, $I^d_{11} = I^d_{22} = \frac{1}{2}I^d_{33} = 1$ (so that $\sigma = 0.42857$ and $\lambda = 0.30984$), $p' = 0.74231$, $\zeta' = 0.3$, $J = 10$, $c^\gamma = 1$, $k^\gamma = 0$.

For the system described above, we have: $\sigma_1 = 0.73841$ rad/sec, and $\sigma_2 = 0.11873$ rad/sec. The two periods associated with σ_1 and σ_2 are: $\tau_1 = 8.50907$ sec, and $\tau_2 = 52.91995$ sec. The time period selected for integrating and averaging $\dot{E}_R$, using Eq. (39) with $N = 1$, is $\tau' = \tau_2 = 52.91995$ sec.

A stable time constant of 133 min was predicted using Eqs. (45) and (47). The time simulation result, shown in Fig. 5, indicates a stable time constant of approximately 120 min. The time history of the ring damper tilting motion β_1 is shown in Fig. 6. The two driving frequencies, σ_1 and σ_2, are distinctly evident in the figure.



NUTATION ANGLE = THE ANGLE BETWEEN P_3 AND THE SPACECRAFT ANGULAR MOMENTUM VECTOR.

Fig. 2 Case I nutation angle time history.

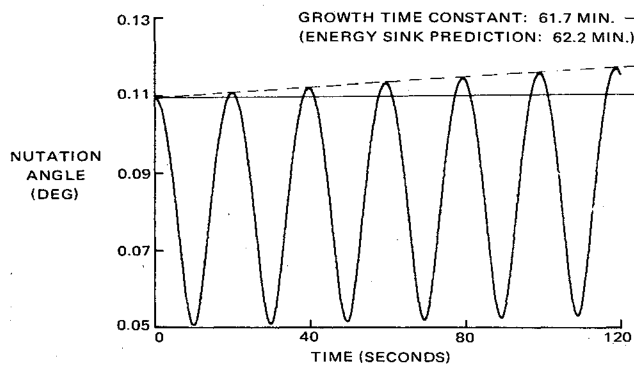


Fig. 3 Case II nutation angle time history.

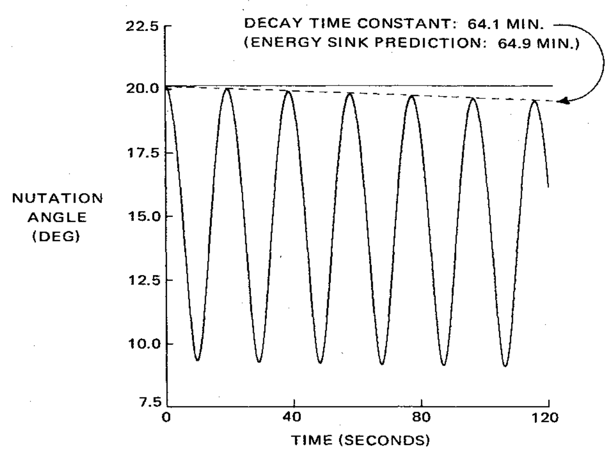


Fig. 4 Case III nutation angle time history.

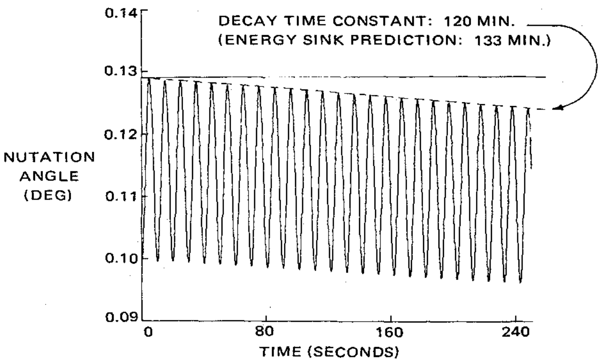


Fig. 5 Case IV nutation angle time history.

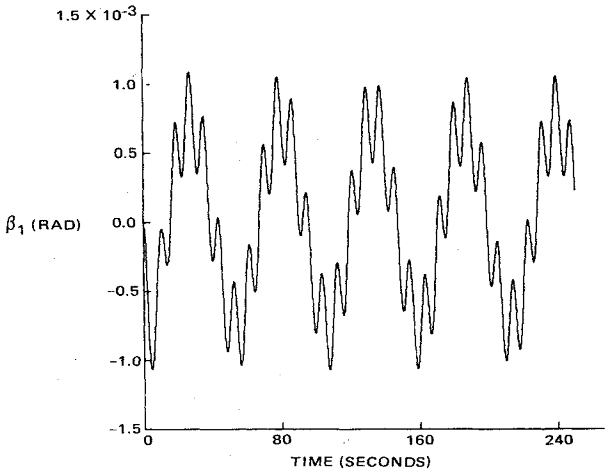


Fig. 6 Case IV rotor damper tilting motion, β_1.

Conclusion

For a symmetric dual-spin spacecraft in a motion state in the immediate neighborhood of its nominal spin, both platform and rotor dampers are excited by only a single frequency, but of different magnitude. In the case of an asymmetric dual-spin spacecraft, the platform damper still responds to a single excitation frequency. The rotor damper, however, is driven by harmonic functions of two different frequencies. Recognizing this phenomenon, the energy dissipation rates can be evaluated following the traditional analytical procedure. Substitution of these energy dissipation rates into the familiar stability condition results in closed-form representations of the nutational stability criterion and the time constant. The numerical examples have demonstrated that the energy-sink predictions correlate with the time simulation results for a class of asymmetric dual-spin spacecraft whose geometric mean of the two transverse inertias does not differ appreciably from the arithmetic mean. The difference between the two transverse inertias, however, can be significant. it is worth noting that most of the real, modern dual-spin spacecraft belong to this class.

One of the numerical examples shows an excellent correlation, even for a nutation angle as large as 20 deg. For all practical purposes, this level of 20 deg is sufficient to cover the range of interest in the spacecraft design for its in-orbit conditions. Invaluable information concerning the spacecraft nutational stability has been, and will continue to be, extracted from energy-sink approach especially in dealing with nonlinear dampers; however, caution must be exercised in its application and the interpretation of its result without a full appreciation of the assumptions and approximations that have been introduced into the analytical process. The energy-sink approach is approximate in nature. More rigorous means, such as system time simulations or other formal methods, should be employed to verify its predictions.

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